Bi-elastic F-Probabilities

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Abstract

We want to convince the audience that “bi-elasticity” is the main functional concept when dealing with “F-probabilities” (Weichselberger) = “coherent probabilities” (Walley).

In the first part we start out from Buja’s “pseudo-capacities” (1986), comprising the neighbourhood models for classical probabilities commonly used in robust statistics: If \( p \) is a classical probability and \( f: [0; 1] \rightarrow [0; 1] \) with \( f(0) = 0 \) and \( f(1) = 1 \) is convex, then \( L = f \circ p \) is the lower bound (lb.) of a 2-monotone capacity. We show that robust statistics can be freed from the severe restriction to 2-monotonicity by employing the more natural framework of F-probabilities, where the main tool for doing this is to use a generalization of convex functions, namely bi-elastic functions \( f \) (= “star-shaped” at 0 and 1). In addition we prove the following closure property: If \( f \) is bi-elastic and \( L_1 \) is the lb. of an F-probability, then \( L = f \circ L_1 \) is the lb. of an (wider) F-probability too; and bi-elasticity is the weakest condition having this closure property. Moreover we point out some generalizations of this approach leading to questions about conditional interval probability.

In the second part we report some work in progress concerning a more-dimensional version of bi-elasticity (MBE): If \( f: [0; 1]^k \rightarrow [0; 1] \) is MBE and \( L_1, \ldots, L_k \) are lb.s of F-probabilities, then \( L = f(L_1, \ldots, L_k) \) is the lb. of an F-probability too; and MBE is the weakest condition having this closure property. MBE has an amazing consequence: If \( f \) is MBE, then its restriction \( f_0 \) to \( \{0, 1\}^k \) is itself the lb. of an F-probability on a k-dimensional measurable space. Moreover, the lb. \( f_0 \) of any finite-dimensional F-probability can be generated in this manner. The other way around, \( f \) is an extension of \( f_0 \) which is wider than Walley’s natural extension.