GENERAL EQUILIBRIUM THEORY AND INCREASING RETURNS

Presentation

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The common characteristic of the papers presented in this Issue is that they all consider a framework with the generality of the Walrasian model, but in which no convexity assumption is made regarding the individual or the aggregate production sets. This allows for increasing returns but also for more general types of 'non-convexities' in production. We present the results obtained in the papers of this Issue, which are mainly concerned with the following questions: (i) behavior of the producer in the presence of increasing returns, (ii) existence of equilibria in a disaggregated framework, and (iii) marginal cost pricing.

1. Introduction

The presence of increasing returns to scale in production sectors such as electricity, railways, etc., and the failure of the competitive mechanism in such an environment are widely recognized in the economics literature. However, attempts to incorporate increasing returns, or to search for alternative mechanisms, in models with the generality of the Walrasian model are only recent; a notable exception is Scarf (1986a) which was written in 1963.

In the past ten years, several authors have made part of general equilibrium theory various topics related to increasing returns. The papers gathered in this Special Issue, which contribute to this endeavor, are mainly concerned with the following questions, which we have classified into three general categories. At the end of this presentation, we shall mention other contributions to topics not examined here.

(i) Behavior of the producer in the presence of increasing returns to scale. It is generally recognized that the standard behavioral assumption of profit maximization may be inapplicable in the presence of increasing returns to scale. The search for alternative rules of behavior, however, is very rarely investigated and the two rules of behavior which are the most often considered are marginal and average cost pricing (and within the latter rule,
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Boiteux–Ramsey and Aumann–Shapley pricing). The papers in this issue consider the formalization of these two rules at a high level of generality, and also propose new rules for the producer in the presence of increasing returns to scale.

The papers by Dehez and Drèze propose two versions of quantity-taking behavior which are both related to the notion of voluntary trading; Dierker and Neuefeind extend the model of Dierker et al. (1985) to add minimal quantity targets for the production sector.

(ii) Existence of equilibria. Soon after it was posed by Guesnerie (1975), the existence problem was considered for marginal cost pricing equilibria (cf. below) by Beato (1979, 1982), Mantel (1979) and then by Brown and Heal (1982) and Cornet (1982). The common feature of these papers is that they exhibit marginal cost pricing equilibria which are aggregate productive efficient under the assumption that the total production set $Y$ has a smooth boundary (or, in the last paper, the weaker assumption that $Y$ has no ‘inward kinks’). After it was shown to be very restrictive by Beato and Mas-Colell (1983, 1985), this type of assumption was ruled out in the following papers which are concerned with the so-called ‘disaggregated approach’. In Dierker et al. (1985), an existence result is provided in a model which distinguishes a priori between inputs and outputs and in which the producers follow special pricing rules. In Beato and Mas-Colell (1985), Bonnisseau and Cornet (1985), and Brown et al. (1986), existence results are provided for marginal cost pricing equilibria.

In this issue, the papers by Bonnisseau, Bonnisseau–Cornet, Kamiya, and Vohra are concerned with the existence of equilibria in a disaggregated framework, where no distinction is made a priori between the inputs and the outputs of the firms; each firm follows a general rule which may be, in particular, profit maximization, marginal or average cost pricing, or one of the rules proposed in (i).

(iii) Marginal cost pricing. The marginal cost pricing doctrine, elaborated in the thirties and forties by (amongst others) Allais, Hotelling, Lange, Lerner, Pigou, and with a long history dating back to Dupuit, was until recently mainly concerned with welfare properties. To quote Hotelling, an optimum of welfare ‘corresponds to the sale of everything at marginal cost’. Statements of this second welfare theorem in a general equilibrium model have been provided, starting with the seminal paper by Guesnerie (1975), and, more recently, by Khan and Vohra (1985, 1987), Yun (1984), Quinzii (1986), Cornet (1986, 1988), and Cornet and Rockafellar (1988). As pointed out by Guesnerie (1975) and by Brown and Heal (1979) [cf. also Beato and Mas-Colell (1985), Vohra (1987)], in the absence of convexity assumptions, the analogue of the first welfare theorem need not hold, in general, and even
none of the marginal cost pricing equilibria may not be productive, unless
one considers a specific model as in Dierker (1986, 1987) and Quinzii (1988).
In this issue papers by Bonnisseau–Cornet, Cornet, Jouini, and Kamiya
are devoted to different questions related to marginal cost pricing which, in
this issue, is formalized by means of Clarke’s normal cone (or the related
concept of generalized gradient). This allows the use of new mathematical
techniques, developed during the past ten years and presented in the books

2. The general equilibrium model

The common characteristic of the papers presented in this issue is that
they all consider a framework with the generality of the Walrasian model, as
presented in Debreu’s Theory of Value or in Arrow and Hahn’s General
Competitive Analysis, but in which no convexity assumption is made
regarding the individual or the aggregate production sets. This allows for
increasing returns to scale but also for more general types of ‘non-
convexities’ in production.1 We shall describe below a model which retains
the essence of those considered in this issue, and to which the reader may
refer for various generalizations. There are positive finite numbers, l, of
goods, m, of consumers and, n, of producers.

An equilibrium of the whole economy is a list of consumption plans \( x_i^* \)
\((i=1,\ldots,m)\), a list of production plans \( y_j^* \) \((j=1,\ldots,n)\), and a non-zero price
vector \( p^* \) in \( R^l \) such that (a) the markets for all goods clear;2 (b) the
consumers maximize their preferences subject to their budget constraints, (c)
every producer \( j \) is in equilibrium at \( (y_j^*, p^*) \). The nature of the equilibrium
condition (c) is the main difference with the Walrasian model and it will be
defined formally below. It should be emphasized that the price vector \( p^* \) is
the same for each producer (and each consumer). An element \( (y_1^*, \ldots, y_n^*, p^*) \)
satisfying condition (c) is called a production equilibrium.

We first describe the production side of the economy. The technological
possibilities of the \( j \)th producer are represented by a subset \( Y_j \) of the
commodity space \( R^l \) and the two following basic assumptions are made:

**Assumption P.** For all \( j \), \( Y_j \) is closed, \( Y_j \) contains 0 (possibility of inaction),
\( Y_j - R^l \subset Y_j \) (free-disposal).

1 In fact, in this issue, only the second paper by Dehez and Drèze will assume explicitly that
increasing returns to scale prevail for some firm \( j \), that is, \( y \in Y_j \) and \( \lambda > 1 \) imply \( \lambda y \in Y_j \).
2 Formally, in this presentation (denoting by \( \omega_h \) the total endowment of good \( h \)), for every \( h \),
\( p_h \geq 0, \sum_{h=1}^h x_h = \sum_{h=1}^h y_h + \omega_h \) with equality whenever \( p_h > 0 \). In some of the papers, this notion
is called a free-disposal equilibrium to distinguish it from the stronger notion of equilibrium for
which it is assumed that \( \sum_{h=1}^h x_h = \sum_{h=1}^h y_h + \omega \).
Assumption B. \( A(\sum_{j=1}^{n} Y_j) \cap -A(\sum_{j=1}^{n} Y_j) = \{0\} \) (asymptotic irreversibility assumption).

It should be noted that Assumption P implies that, for every \( j \), the boundary, \( \partial Y_j \), of the production set \( Y_j \) is exactly the set of (weakly) efficient production plans of the \( j \)th producer.

The behavior of the \( j \)th producer is then described by its supply correspondence, \( \eta_j \), which associates with each price vector \( p \) in \( R^l \), a subset \( \eta_j(p) \) of efficient production plans; thus \( \eta_j \) is a correspondence from \( R^l \) to \( \partial Y_j \). Given \( \eta_j \), the equilibrium condition (c) of the \( j \)th producer at the pair \((y_j^*, p^*) \in \partial Y_j \times R^l \) is then defined by \( y_j^* \in \eta_j(p^*) \). There is an alternative approach to describe the behavior of the producer with the notion of a pricing rule. If the supply correspondence, \( \eta_j \), is taken as the primitive concept, one associates the inverse correspondence \( \phi_j \) of \( \eta_j \), i.e., the correspondence from \( \partial Y_j \) to \( R^l \), defined by \( \phi_j(y_j) = \{ p \in R^l | y_j \in \eta_j(p) \} \); \( \phi_j \) is called the pricing rule of the \( j \)th producer and the above equilibrium condition can be equivalently rewritten by saying that \( p^* \in \phi_j(y_j^*) \). If the pricing rule \( \phi_j \), a given correspondence from \( \partial Y_j \) to \( R^l \), is taken as the primitive concept, then it is the inverse of a unique supply correspondence. Thus, logically, the two approaches are equivalent. In the Walrasian model, and in the monopolistic model described below, the primitive concept is the supply correspondence. As we shall see, however, in the absence of convexity assumptions, the pricing rule correspondence may satisfy continuity and convex valuedness properties when the supply correspondence does not. This is the main reason for which all the papers in this issue consider only the notion of a pricing rule. The economic interpretation of these two notions is discussed below, or within each specific model in the papers of this issue. As yet, we only point out that the pricing rule (or the supply correspondence) may be either endogenous or exogenous to the model, and that it allows both price-taking and price-setting behaviors.

The above framework is compatible with various behaviors which are currently considered in the economics literature. Firstly, if the \( j \)th producer is maximizing profits, its supply at a non-zero price vector \( p \) is standardly defined by \( \eta_j(p) = \{ y_j \in Y_j | p \cdot y_j \geq p \cdot y \text{ for all } y \in Y_j \} \) (which we notice is a subset of \( \partial Y_j \)), and we let \( \eta_j(0) = \partial Y_j \). Other examples considered in this issue are average or marginal cost pricing rules, or voluntary trading for which the pricing rules are defined for producer \( j \), respectively, by:

\[
AC_j(y_j) = \{ p \in R^l_+ | p \cdot y_j = 0 \},
\]

\( A(\sum_{j=1}^{n} Y_j) \) denotes the asymptotic cone of the aggregate production set \( \sum_{j=1}^{n} Y_j \) and we point out that Assumption B is satisfied if \( \sum_{j=1}^{n} Y_j \) is closed, convex, contains 0 and if the standard irreversibility assumption holds, i.e., \( \sum_{j=1}^{n} Y_j \cap -\sum_{j=1}^{n} Y_j = \{0\} \).
\[ MR_j(y_j) = N_Y(y_j) \] Clarke's normal cone to \( Y_j \) at \( y_j \) (cf. section 5),

\[ VT_j(y_j) = \{ p \in R^l \mid \langle p, y_j \rangle \leq \langle p, y \rangle \text{ for all } y \in Y_j \text{ such that } y \leq y_{j+} \}. \]

The marginal cost pricing doctrine is generally studied in the context of a public monopoly and is associated with normative theory; section 5 is devoted to it. The budgetary constraints (average cost pricing and also the so-called Boiteux–Ramsey pricing) have also been proposed for the management of public monopolies; this is also the main interpretation of the non-convex firms in the paper by Dierker and Neuefeind. Voluntary trading is considered in the two papers by Dehez and Drèze who are concerned with the positive theory of decentralized equilibria in terms compatible with increasing returns. In the model of monopolistic competition, as presented in Arrow and Hahn (1971), the behavior of a monopolistic producer is described by its supply, \( \eta_j \), which is assumed to be a continuous function. However, the continuity assumption on \( \eta_j \) [or the weaker assumption, made by Silvestre (1977a, b, 1978), that \( \eta_j \) is a correspondence with a closed graph and non-empty closed convex values] is quite restrictive in such a model, as pointed out by Roberts and Sonnenschein (1977). Moreover, in the absence of convexity assumptions, the continuity assumption (or its weaker form) is not satisfied, in general, by the supply correspondences associated with the pricing rules \( AC_j, MR_j \), and \( VT_j \). Finally, the existence results of Bonnisseau–Cornet, Kamiya, and Vohra allow for general pricing rules which are assumed to satisfy the following continuity property

**Assumption PR.** For all \( j, \phi_j \) has a closed graph, and, for all \( y, \phi_j(y) \) is a closed convex cone with vertex 0 in \( R^l \), and which is not reduced to \( \{0\} \).

It is worth pointing out that the three pricing rules \( AC_j, MR_j \) and \( VT_j \) satisfy Assumption PR if Assumption P holds and if \( Y_j \cap R^l_+ = \{0\} \) (impossibility of free production). Furthermore, if the producer \( j \) is maximizing

\[ \bar{y}_j \] denotes the vector in \( R^l \) with coordinates \( \max \{0, y_{jh}\} (h = 1, \ldots, l) \). Note that voluntary trading implies, in particular, cost minimization.

\footnote{In their model, the supply \( \eta_j(x_j, (y_j), p) \) of a monopolistic producer \( j \) is assumed to depend not only upon the price vector but also upon the production plan of each producer and the consumption plan of each consumer. The above model can easily be generalized to allow \( \eta_j \) and also \( \phi_j \) for such a dependence and, in fact, this is partly done by some of the papers of this issue. The main difference between the two models comes from the fact that, in this issue, the behavior of the producer embodies the normative requirement, that its production is efficient. This condition may not be satisfied in models of imperfect competition and, in Arrow and Hahn, the supply \( \eta_j(x), (y_j), p \) is only assumed to belong to the production set \( Y_j \). Although the analysis of the strategic behavior of the producers in the presence of increasing returns to scale is a fundamental question, it will not be considered by the papers of this issue.}
profits, its associated pricing rule also satisfies Assumption PR if $Y_j$ is convex
and if Assumption P holds.

The consumption sector of the economy is standard, except for its
distributional side when the producers exhibit deficits. The $i$th consumer
($i = 1, \ldots, m$) is characterized by his (her) consumption set $X_i$, a non-empty,
closed, convex, bounded below subset of $R^l$, his (her) preference relation $\preceq_i$,
a complete, continuous, convex, non-satiated, preorder on $X_i$, his (her) initial
endowment $\omega_i$, a vector in $R^l$, and his (her) revenue function $r_i: \prod_{j=1}^n Y_j \times R^l \to R$. The above assumptions (C) on $X_i$ and $\preceq_i$ are the same as in
Debreu's *Theory of Value*, this is also the case for the revenue functions $r_i$ in
the important case where the pricing rules of the producers precludes negative profits, i.e., they satisfy the loss-free property.

**Assumption LF.** For all $j, p \in \Phi_j(Y_j)$ implies $p \cdot y_j \geq 0$.

Under Assumption P, this holds, for example, whenever the producer is maximizing profits, or following the average cost or the voluntary trading
pricing rules. In the loss-free case, one typically considers a private ownership
economy as in the Walrasian model; thus, the wealth of the $i$th consumer is
$r_i(y_1, \ldots, y_n, p) = p \cdot \omega_i + \sum_{j=1}^n \theta_{ij} p \cdot y_j$, where $\theta_{ij} \geq 0$ is the share of
the $i$th consumer in the profit of the $j$th producer and it is assumed that, for
every $j$, $\sum_{i=1}^n \theta_{ij} = 1$. However, when the producers may suffer losses (as, for example, a public firm following the marginal cost pricing rule), we need to
allow for more general revenue structures in order to finance the deficits of
the firms. We shall assume the following basic properties.

**Assumption R.** For every $i$, the revenue function $r_i: \prod_{j=1}^n Y_j \times R^l \to R$ is
continuous, positively homogeneous of degree one with respect to the price
vector, and satisfies $\sum_{j=1}^n r_i(y_1, \ldots, y_n, p) = p \cdot (\sum_{j=1}^n y_j + \omega)$, where $\omega = \sum_{i=1}^m \omega_i$ denotes the total initial endowment of the economy.

As yet we have not specified which variables (prices or quantities) each
producer takes as given and which ones it chooses. The model is in fact quite
flexible. It allows price-taking behavior and it encompasses, in particular, the
Walrasian model, as shown earlier. It also allows price-setting behavior, as
we shall now see with the model of Dierker, Guesnerie and Neuefeind (1985),
which is a particular case of the model described above.

In their model, called DGN below, it is assumed that for each producer
one can distinguish a priori between inputs and outputs. Thus, for every $j$,
there is a partition of the set of goods $\{1, \ldots, l\}$ in two non-empty subsets $I_j$.
and $O_j$ such that, up to free-disposal, for each producer $j$ the goods in $I_j$ are inputs and the goods in $O_j$ are outputs. Furthermore, in the DGN model, two different producers do not produce the same good, i.e., $O_j \cap O_{j'} = \emptyset$, whenever $j \neq j'$. Each producer $j$ takes as given the input prices $(p_h, h \in I_j)$ and the output quantities $(y_{jh}, h \in O_j)$; each is instructed (i) to choose the input quantities $(y_{jh}, h \in I_j)$ by minimizing its cost, and (ii) to set the prices of its outputs $(p_h, h \in O_j)$ according to a special pricing rule $\psi_j$ which depends upon the price vector $p$ and the output quantities, i.e., $\psi_j$ is a correspondence from $R^I_+ \times R^O_{+j}$ to $R^O_{+j}$, where $R^O_{+j} = \{ p \in R^I \mid p_h = 0 \text{ if } h \notin O_j \}$ and, for $p \in R^I$, we denote by $p^O_j$ the projection of $p$ onto $R^O_{+j}$, i.e., $p^O_{hj} = p_h$ if $h \in O_j$, and $p^O_{hj} = 0$ if $h \notin O_j$. Then the behavior of the $j$th producer can be formalized within the above framework by defining the pricing rule $\phi_j$ to be the correspondence from $\partial Y_j$ to $R^I$.

$$\phi_j(y_j) = \begin{cases} p \in R^I_+ & \text{if } \begin{array}{l} n \cdot y_j \geq n \cdot y \text{ for all } y \in Y_j \text{ such that } y_+ = y_j, \text{ and} \\ p^O_j \in \psi_j(p, y^O_{+j}) \end{array} \\ (i) \\ (ii) \end{cases}$$

Thus, an equilibrium in the DGN model is an element $((x^*_i), (y^*_j), p^*)$ satisfying the conditions (a), (b), (c), with $\phi_j$ defined as above, and the additional requirement that $y^*_j \geq 0$ for every $j$.

3. Existence results for general pricing rules

The contributions by Bonnissseau-Cornet, Kamiya, and Vohra to this issue provide results on the existence of equilibria in the model described above with general pricing rules. In addition to Assumptions P, B, PR, C and R, each paper makes different assumptions which are not directly comparable, except for the so-called survival assumption (Assumption SA) which is common to the three papers. This assumption asserts that, at every $(y_1, \ldots, y_n, p)$ in the set $PE$ of production equilibria (or a subset of it for Vohra), the consumer can 'survive', in the sense that their revenues are above their subsistence levels, i.e., $r_i(y_1, \ldots, y_n, p) > \inf p \cdot X_i$ for every $i$. It should be noted that in a private ownership economy in which the pricing rule of each producer is loss-free, Assumption SA is implied by the standard survival assumption, namely, $\omega_i \in X_i + \text{int } R^I_+$ for every $i$, which is usually made for the existence of Walras equilibria [cf. Debreu (1959)]. At this stage it is

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7I.e., for every $y \in Y_j$, there exists $\tilde{y} \in Y_j$ such that $\tilde{y} \geq y$, $\tilde{y}_h \leq 0$ for every $h \in I_j$, and $\tilde{y}_h \geq 0$ for every $h \in O_j$. In fact we only present here a simplified version of the DGN model, in which no free-disposal assumption is made and which allows for a convex production sector without an a priori distinction between inputs and outputs.

8This implies that the total wealth satisfies $p \cdot (\sum_{j=1}^n y_j + \omega) > \inf p \cdot \sum_{i=1}^n X_i$ for every production equilibrium $(y_1, \ldots, y_n, p)$. This weaker property is called the survival assumption by some authors in this issue.
worth pointing out that Assumptions P, B, PR, C, R and SA in themselves do not guarantee, in general, the existence of equilibria (cf. Remark 2.6 of the first paper) unless one considers particular pricing rules such as marginal cost pricing (cf. section 5) or loss-free pricing rules.

We now present the additional assumptions which are made in the three papers to guarantee the existence of equilibria. In the Bonnisseau–Cornet paper, it is assumed in addition that the pricing rule of each producer has bounded losses, i.e.,

Assumption BL. For every \( j \), for some real number \( \alpha_j \), \( p \in \phi_j(y_j) \cap S \) implies \( p \cdot y_j \geq \alpha_j \) (where \( S \) denotes the simplex of \( R^d \)).

Furthermore, it is shown that for the marginal (cost) pricing rule, Assumption BL is equivalent to the star-shapedness of the production sets \( Y_j \). Kamiya combines a stronger assumption than Assumption B, with a weaker assumption than Assumption BL; namely, he assumes that \( \co A(\sum_{j=1}^n Y_j) \setminus -\co A(\sum_{j=1}^n Y_j) = \{0\} \) and that, for every \( j \), for every sequence \( (y_j^{(v)}, p^{(v)}) \) in \( \partial Y_j \times S \) such that \( \|y_j^{(v)}\| \to +\infty \), and \( p^{(v)} \in \phi_j(y_j^{(v)}) \) for every \( v \), one has \( \liminf_{v \to +\infty} p^{(v)} \cdot (y_j^{(v)}/\|y_j^{(v)}\|) \geq 0 \). Vohra makes a stronger assumption than Assumption SA but on a subset of \( PE \). As Vohra states, ‘given reference prices and production plans which are not too far from the attainable production sets, if the income of any consumer becomes non-positive\(^9\) the pricing rule of some firm instructs it to raise the price of some output above its reference price or to lower the price of some input …’.

The contribution of Bonnisseau generalizes the existence result of Dierker et al. (1985) in the DGN model described previously and also provides an alternative proof of the existence result by Kamiya in this issue. These two results are deduced from the above existence result of Bonnisseau and Cornet after modifying the production side of the economy outside the attainable production sets. The proofs of the existence results of Bonnisseau–Cornet and Vohra rely essentially on Kakutani’s theorem; this is also the case for Bonnisseau’s proofs, whereas the original one of Dierker et al. (1985) relied on Eilenberg–Montgomery’s fixed-point theorem. In contrast, Kamiya uses degree theory for correspondences; this allows him to also provide conditions guaranteeing the uniqueness of equilibria.

4. Quantity-taking behavior and quantity targets for the firm

The first paper by Dehez and Drèze proposes the following equilibrium conditions for the producer (i) voluntary trading: at the prevailing prices,

\(^9\)Vohra makes the additional assumption that, for every \( i \), \( X_i \subset R^d \), and \( 0 \in X_i \). Without this assumption, one should replace ‘becomes positive’ by ‘less than or equal to inf \( p \cdot X_i \)’.
profit cannot be increased by reducing its outputs or by choosing a different input combination and (ii) minimality of the output prices: lower output prices will not sustain the same output quantities. The first notion is formalized, for the jth producer, by the pricing rule $VT_j(yj)$ defined previously by $VT_j(yj) = \{ p \in R^l | p \cdot yj \geq p \cdot y \}$ for every $y \in Y_j$ such that $y \leq yj + \epsilon$, i.e., the set of price vectors $p$ such that it is profitable for the producer to meet the demand as given by $yj + \epsilon$, instead of producing less. The minimality condition is formally defined by requiring that the price vector $p$ belongs to the following subset of $VT_j(yj)$:

$$\psi_j^*(yj) = \{ p \in VT_j(yj) | \exists p' \in VT_j(yj), p' \leq p, p' \neq p, \text{ and } p' = p \text{ for } h \in I_j(yj) \},$$

where

$$I_j(yj) = \{ h = 1, \ldots, l | yjh < 0 \text{ or } yjh \leq 0 \text{ for all } y_j \in Y_j \},$$

that is, the set of price vectors $p$ in $VT_j(yj)$ whose output components are 'minimal' subject to voluntary trading. Then the pricing rule of the jth producer is defined to be the smallest correspondence $\psi_j^*$ from $\partial Y_j$ to $R^l$ satisfying Assumption PR and containing $\psi_j^*$, together with an additional requirement in the degenerate case when $\psi_j^*(yj) = \{ 0 \}$. In their paper, existence of equilibria is proved for a private ownership economy in which the producers follow the behavior described above (Theorem 2). It is also shown that, whenever the production sets are convex, their equilibrium concept coincides with the competitive one (Theorem 1).

In their second paper, Dehez and Drèze study the properties of a class of production sets $Y_j$ which exhibit increasing returns to scale and which is directly related to the notion of voluntary trading. This class of production sets, called output-distributive, is defined as the analogue on the output side of the notion of input distributive sets introduced by Scarf (1986a) in his study of the non-emptiness of the core in the presence of increasing returns. Here, Dehez and Drèze show that, for an output-distributive set $Y_j$, average cost pricing is compatible with voluntary trading (Proposition 1), and also provide a converse of this assertion (Theorem 1). Formally, if $Y_j$ is output-distributive and satisfies Assumption P, then the pricing rule $\phi_0$ defined by $\phi_0(yj) = VT_j(yj) \cap AC_j(yj)$ satisfies Assumption PR; hence, in particular, for all $y_j \in \partial Y_j$, $\phi_0(yj) \neq \{ 0 \}$. This latter property essentially characterizes the output-distributive production sets. This class of production sets is also related to the notions of supportable cost functions. Furthermore, a convex production set is output distributive if and only if it is a cone. Together with the zero-profit property, this remark suggests that output-distributive sets provide the natural non-convex counterpart to convex cones.
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In this paper, Dehez and Drèze also provide an existence result (Theorem 2) for a private ownership economy in which the producers follow the behavior defined by average cost pricing and voluntary trading, i.e., the pricing rule $\phi_o$.

In this issue Dierker and Neuefeind extend the DGN model. They add to it minimal quantity requirements, supposing, as they express it, that 'there is a planning agency that ensures that a certain net production of the outputs is achieved'. Formally, the minimum quantity targets are defined by functions $b_h: \prod_{j=1}^n R_{Oj}^o \times R_+^t \to R$, for $h \in \bigcup_{j=1}^n O_j$, and, as in the DGN model, the special pricing rule of the $j$th producer is a correspondence $\psi_j$ from $R_{Oj}^o \times R_+^t$ to $R_{Oj}^o$. An equilibrium in this model is now an element $((x_1^*), (y_1^*), p^*)$ satisfying $y_{Oj}^* \geq 0$ for all $j$, and the above conditions (a) and (b), with (c) replaced by the three following conditions, for every $j$:

(i) (cost minimization) $p^* \cdot y_+^* \geq p^* \cdot y$ for all $y \in Y_j$ such that $y_+ = y_{+j}^*$;
(ii) (special pricing rule) $p_{Oj}^* \leq q_j^* \in \psi_j(y_{Oj}^*, p^*)$;
(iii) (quantity target) for every $h \in \bigcup_{j=1}^n O_j$, $\sum_{j=1}^n y_h^* \geq b_h(y_{Oj}^*, \ldots, y_{O_n}^*, p^*)$, and if the inequality is strict for some $h \in O_j$, then $p_h^* = q_h^*$.

In other words, each producer $j$ (i) minimizes its cost, and (ii) sets the prices of its outputs below the level suggested by the pricing rule $\psi_j$; (iii) the production aspirations of the planning agency are satisfied and if the target is exceeded for some good, then the price of the output charged by the producer which produces it is then equal to the level suggested by the pricing rule. As pointed out by Dierker and Neuefeind, 'one cannot expect, in general, that the pricing rule applies at the market price [i.e., that $p_{Oj}^* \leq \psi_j(y_{Oj}^*, p^*)$ in (ii)] since minimal targets may be set too high to allow for the fulfillment of the pricing rule'. In their paper, Dierker and Neuefeind state assumptions which guarantee the existence of such an equilibrium.

5. Marginal (cost) pricing

The marginal cost pricing doctrine is usually presented in a model à la Dierker, Guesnerie and Neuefeind in which one can distinguish a priori between the inputs and the outputs of the producers. Then the producer is instructed (i) to choose the level of its inputs by minimizing its cost and (ii) to set the prices of its outputs according to its marginal costs, taking as given the 'dual' variables, i.e., the market prices of its inputs and the output levels.

In the absence of an a priori distinction between inputs and outputs, the $j$th producer is said to follow the marginal profit pricing rule, or simply the
marginal pricing rule\textsuperscript{10} if its equilibrium condition at every pair \((y_j, p) \in \partial Y_j \times R^l\), requires the fulfillment of the 'first-order necessary condition for profit maximization over \(Y_j\), i.e., formally if \(p\) belongs to \(N_{Y_j}(y_j)\), the normal cone to \(Y_j\) at \(y_j\) in the sense of Clarke (1975)\textsuperscript{11}. In other words, this formalization provides a generalization of the notion of marginal rates of transformation for the producer in the absence of smoothness and convexity assumptions (and in fact for an arbitrary production set \(Y_j\)). A fundamental reason for the choice of Clarke's normal cone, among the many notions encountered in the literature, is due to the fact that it satisfies Assumption PR if Assumption P holds [cf. Cornet (1982) and Rockafellar (1979)].

The two papers by Cornet and Kamiya investigate the consequences of Assumption SA in an economy where each producer follows the marginal pricing rule, in which case the equilibrium notion is called a marginal pricing equilibrium. They consider the two following questions which are directly related: (i) existence or non-existence of marginal pricing equilibria, and (ii) topological properties of the sets

\[
A(\omega) = \left\{ (y_j) \in \prod_{j=1}^{n} Y_j \left| \sum_{j=1}^{n} y_j + \omega \in R^l_+ \right. \right\}, \text{ resp.}
\]

\[
EA(\omega) = \left\{ (y_j) \in \prod_{j=1}^{n} \partial Y_j \left| \sum_{j=1}^{n} y_j + \omega \in R^l_+ \right. \right\},
\]

of attainable\textsuperscript{12} (resp. efficient) production plans. The topological properties of the \(\varepsilon\)-neighborhoods of the set \(EA(\omega)\) play a fundamental role in the paper by Bonnisseau and Cornet (1985) where the existence of marginal pricing equilibria is proved under Assumptions P, B, C, R, SA, and the additional assumption that each production set has a 'smooth boundary'.

The second paper by Kamiya shows that, in general, there may not exist marginal pricing equilibria in the absence of the survival assumption, even under the assumption that \(\omega_i \in X_i + \text{int } R^l_i\) for every \(i\). His counterexample

\textsuperscript{10}In the absence of convexity assumptions, a producer following this rule may not be minimizing his cost; cf. Arrow and Hurwicz (1960), Guesnerie (1984). Thus, it is somewhat misleading to call this notion marginal cost pricing as it is usually done in the literature and we shall reserve the term 'marginal cost pricing' to cases for which the cost function is defined and is in fact minimized by the producer.

\textsuperscript{11}We point out two cases of particular interest. Firstly, if \(Y_j\) is convex, then the marginal pricing rule coincides with profit maximization. Secondly, if \(Y_j\) has a 'smooth' boundary, i.e., if \(\partial Y_j\) is a \(C^2\) submanifold in \(R^l\), then under Assumption P, \(N_{Y_j}(y_j)\) is the closed half-line of outward normal vectors to \(Y_j\) at \(y_j\) and is a subset of \(R^l_+\); thus, for some good \(h\), say \(h = 1\), one has \(p_1 > 0\) for all \(p \in N_{Y_j}(y_j) \setminus \{0\}\). If we choose the first good as numéraire, then the condition \(p \in N_{Y_j}(y_j) \setminus \{0\}\) can be equivalently rewritten by saying that, for every \(h = 2, \ldots, l\), \(p_h = MRT_{I_j}(y_j)\), the marginal rate of transformation of producer \(j\), between goods \(h\) and \(1\).

\textsuperscript{12}The word 'attainable' is meaningful when the total consumption set is \(R^l_+\), i.e., \(\sum_{i=1}^{n} X_i = R^l_+\).
consists of an economy with three goods, an arbitrary number \( m \geq 3 \) of consumers with consumption sets \( X_i = R^i_+ \) and two firms such that Assumptions P, B, C, R, and the above smoothness assumption hold. As for the second question (ii), in the absence of Assumption SA the results are quite negative since, in Kamiya's counterexample, the sets \( A(\omega) \) and \( E(\omega) \) are not contractible (but connected), and Arrow and Hahn (1971, p. 156) provide an example where both sets are not connected.

If the survival assumption holds, however, these two sets have 'nice' topological properties, as shown in Cornet's contribution to this issue. More precisely, Cornet shows that, if Assumptions P, B, and SA hold, and if each production set has a 'smooth' boundary, then the set \( A(\omega) \) is homeomorphic to the closed unit ball \( B^m \) of \( R^m \) (Theorem 1') and \( E(\omega) \) is homeomorphic to the closed unit ball \( B^{d-1} \) of \( R^{d-1} \) (Theorem 1). In the case of a single producer (i.e., if \( n = 1 \)), the above results are well known and straightforward; the extension to an arbitrary number of producers, however, relies on elaborate techniques from differential topology.

The most common justification of marginal cost pricing is that a suitable price system and distribution of income allows the decentralization of every Pareto-optimal allocation if each producer is instructed to follow the marginal cost pricing rule. The second paper by Bonnisseau and Cornet provides a version of the second welfare theorem in the line of Guesnerie's work. Their main result states that, under Assumptions C and P, at every Pareto-optimal allocation \( ((x_1^\ast), (y_1^\ast)) \), a non-zero price vector \( p^* \) can be associated such that (a) consumers minimize their expenditures, and (b) producers follow the marginal pricing rule as formalized above, i.e., \( p^* \in N_{Y_j}(y_j^\ast) \) for every \( j \). In fact, the model considered by Bonnisseau and Cornet is more general and allows (i) infinitely many commodities as in Debreu (1954) and Khan and Vohra (1987b), (ii) non-complete, non-transitive preferences, as in Gale and Mas-Colell (1977), and finally (iii) more general rules which encompass the above formalization and the one with Dubovickii–Miljutin's normal cone considered by Guesnerie (1975), which enables them to generalize Guesnerie's result.

Finally, Jouini's paper constructs a production set \( Y \subset R^I \) satisfying Assumption P, and such that Clarke's normal cone is equal to \( R^I_+ \) for every \( x \in \partial Y \) (Theorem 1). Thus, for such a production set \( Y \), the marginal pricing rule does not impose any restriction on the price vectors satisfying the equilibrium condition of the producer. Even if this counterexample may seem pathological on economic grounds, it is important to know that it is not excluded by the theory, and it poses the problem of defining the marginal pricing rule by a set smaller than Clarke's normal cone which would still be 'meaningful'. The existence of the set \( Y \) is deduced by Jouini from the following result (Theorem 1bis), also of interest in itself, which asserts that there exists a Lipschitzian function \( f: R^{d-1} \rightarrow R \) such that, for every \( x \), the
generalized gradient $\partial f(x)$ in the sense of Clarke is equal to the polytope \( \{ p \in \mathbb{R}_+^{l-1} \mid \sum_{k=1}^{l-1} p_k \leq 1 \} \); this theorem extends to several dimensions a previous result by Rockafellar (1981) when $l=2$. A recent result by Jouini (1988) allows to replace the above polytope by an arbitrary non-empty, convex, compact subset of $\mathbb{R}^{l-1}$.

6. Conclusion

In *Theory of Value* (p. 49), Debreu already emphasizes that there are 'three phenomena that the present (general equilibrium) analysis does not cover: (1) *external economies and diseconomies*, i.e., the case where the production set of a producer depends on the production of the other producers (and/or on the consumptions of consumers), (2) increasing returns to scale, (3) the behavior of producers who do not consider prices as given in choosing their productions.'

This situation is beginning to change and research continues to progress in several directions. An account of all the results already obtained, however, goes beyond the scope of this introduction. We mentioned earlier the work on *monopolistic competition* and the seminal paper by Scarf (1986a) on the *non-emptiness of the core*, to which we should add the work by Sharkey (1979), Ichiishi (1980). Quinzii (1982), Ichiishi and Quinzii (1983) and Böhm (1987). We also draw attention to the following topics studied in the presence of increasing returns: *indivisibilities in production*, considered recently in a sequence of papers by Scarf (1981a, b, 1986b), *planning procedures* [cf. Heal (1973), Cremer (1977, 1978), Henry and Zylberberg (1978)], *decentralized resource mechanisms* [cf. Arrow and Hurwicz (1960), Calsamiglia (1977, 1982)], *dynamics* [cf. the recent paper by Romer (1986) and the references in this paper], and the *computation of equilibria* [this latter question was first considered by MacKinnon (1979), in a little known paper which also provides existence results similar to those by Dierker et al. (1985), and more recently by Kamiya (1986, 1987)]. Other contributions are listed in the references which show that the actual body of work on these questions is now impressive. However, much remains to be done and we hope that this Special Issue will encourage further work on this subject.

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